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MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2022/2023 (TERM ID 2210)

ETM4096 - DIGITAL SIGNAL PROCESSING
(TE / RE)

6 FEBRUARY 2023
9:00 a.m - 11:00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of 11 pages (including cover page) with 4 questions and 6-page list of formulas given in the appendix.
2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write all your answers in the Answer Booklet provided.

QUESTION 1

(a) A discrete-time signal $x[n]$ is defined as:

$$x[n] = \begin{cases} n/2, & -2 \leq n \leq -1 \\ 1, & 0 \leq n \leq 2 \\ 0, & \text{elsewhere} \end{cases}$$

Sketch a stem plot for $x[-n+1]\delta[n-2]$

[6 marks]

(b) Let $x[n] = \{1, 2, 3\}$ and $h[n] = \{4, 0, 1, 6\}$ where the arrows indicate the value at $n = 0$.

↑

↑

Find the convolution $y[n] = x[n]*h[n]$ using multiplication method.

[8 marks]

(c) The following is the impulse response of a discrete-time system:

$$h[n] = 5^n u[-n-1]$$

Determine analytically whether the system:

i. is causal

[3 marks]

ii. is stable

[3 marks]

(d) Given a linear constant coefficient difference equation

$$y[n] - 0.5y[n-1] = x[n] + 2x[n-1] + x[n-2]$$

i. Determine the frequency response of an LTI system whose input and output satisfy the difference equation.

[3 marks]

ii. Is the LTI system stable? Justify your answer.

[2 marks]

Continued...

QUESTION 2

An LTI system with system function $H_1(z)$ is given as:

$$H_1(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}$$

(a) Draw the pole-zero plots to check whether the corresponding LTI system is stable and causal, if the ROC is

i. $|z| > 1$

[5 marks]

ii. $|z| < 0.5$

[5 marks]

iii. $0.5 < |z| < 1$

[5 marks]

(b) Realize the system function $H_1(z)$ in direct form II implementation.

[4 marks]

(c) Let $H(z) = H_1(z) + H_2(z)$ and $H_2(z) = \frac{0.2z^{-1}}{1 - 0.4z^{-1} + 0.04z^{-2}}$, realize the system function $H(z)$ in parallel implementation.

[6 marks]

Continued...

QUESTION 3

(a) Figure Q3 shows the desired specification of a digital lowpass filter.

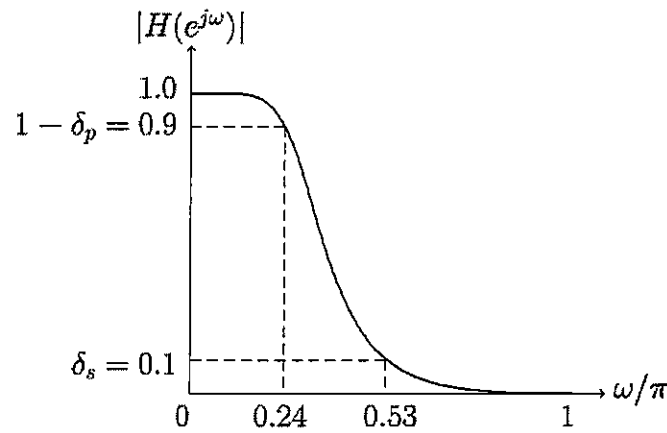


Figure Q3

Using a Butterworth lowpass filter, design a digital IIR filter to satisfy the given specification by using bilinear-transform method. Let $\omega_c = 0.3\pi$ and $T_s = 2s$.

- i. Compute the required order of the filter. [6 marks]
- ii. Determine the system function of the analog prototype filter $H_a(s)$. [6 marks]
- iii. Show that the frequency response of the digital IIR filter $H(z)$ can be written as:

$$H(z) = \frac{0.1323}{\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^3 + 1.019\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + 0.5192\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 0.1323}$$

[2 marks]

- (b) A signal is sampled at the sampling rate of 20 KHz, and has useful signal content in the 0-2 KHz band, but is corrupted with noise in the 5 KHz to 10 KHz band. Use a fixed window (Rectangular, Hanning, Hamming or Blackman), design a digital FIR low-pass filter to attenuate the noise by 42 dB without affecting the signal content by more than 2 dB.

[11 marks]

Continued...

QUESTION 4

Answer the following question based on **your own** student ID (SID) where

$$SID = \{d1, d2, d3, d4, d5, d6, d7, d8, d9, d10\}$$

(a) Figure Q4 illustrates how Discrete Fourier Transform (DFT) is used to perform linear convolution of two finite sequences $x[n]$ and $y[n]$.

Consider $x[n] = \{d2, d4, d6, d8\}$ and $y[n] = \{d7, d9, -2\}$.

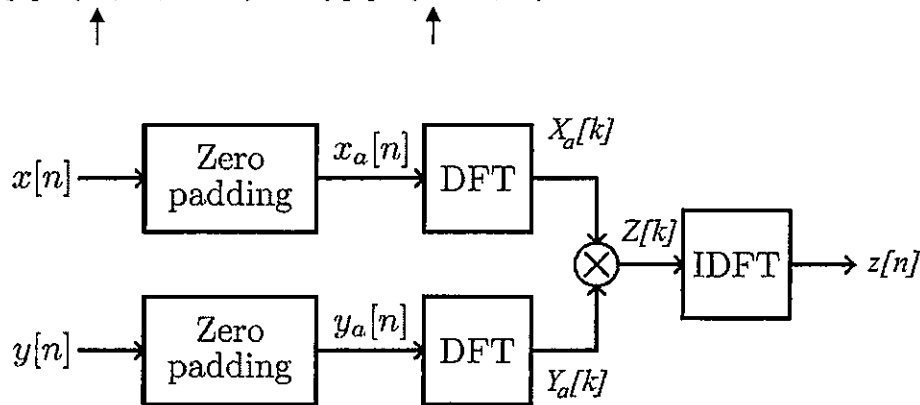


Figure Q4

- i. Use the decimation-in-time Fast Fourier Transform (FFT) algorithm to compute $Y_a[k]$. Draw the butterfly diagram.

[12 marks]

- ii. Verify your answer in (a) with your choice of either direct computation of DFT equation or matrix approach.

[4 marks]

(b) Obtain circular convolution of,

$$x_1[n] = \{d2, d4, d6, d8\} \text{ and } x_2[n] = \{d1, d3, d5, d7\}$$

[9 marks]

End of Questions

APPENDIX

SUMMARY OF USEFUL TABLES

TABLE A1. DISCRETE-TIME FOURIER TRANSFORM PAIRS

Sequence	Discrete-Time Fourier Transform
$\delta[n]$	1
$\delta[n - n_o]$	$e^{-j\omega n_o}$
1 $(-\infty < n < \infty)$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega + 2\pi k)$
$a^n u[n] \quad (a < 1)$	$\frac{1}{1 - ae^{-j\omega}}$
$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega + 2\pi k)$
$(n+1)a^n u[n] \quad (a < 1)$	$\frac{1}{(1 - ae^{-j\omega})^2}$
$\frac{r^n \sin \omega_p (n+1)}{\sin \omega_p} u[n] \quad (r < 1)$	$\frac{1}{1 - 2r \cos \omega_p e^{-j\omega} + r^2 e^{-j2\omega}}$
$\frac{\sin \omega_c n}{\pi n}$	$X(e^{j\omega}) = \begin{cases} 1, & \omega < \omega_c \\ 0, & \omega_c < \omega < \pi \end{cases}$
$x[n] = \begin{cases} 1, & 0 \leq n \leq M \\ 0, & \text{otherwise} \end{cases}$	$\frac{\sin[\omega(M+1)/2]}{\sin(\omega/2)} e^{-j\omega M/2}$
$e^{j\omega_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega - \omega_0 + 2\pi k)$
$\cos(\omega_0 n + \phi)$	$\sum_{k=-\infty}^{\infty} [\pi e^{j\phi} \delta(\omega - \omega_0 + 2\pi k) + \pi e^{-j\phi} \delta(\omega + \omega_0 + 2\pi k)]$

TABLE A2. SOME COMMON z -TRANSFORM PAIRS**The z -transform**Properties of the z -transform

Property	$x[n]$	$X(z)$	$\mathcal{R}_x$
Linearity	$ax[n] + by[n]$	$aX(z) + bY(z)$	$\mathcal{R}_x \cap \mathcal{R}_y$
Time shifting	$x[n - m]$	$z^{-m}X(z)$	$\mathcal{R}_x$
Time reversal	$x[-n]$	$X(z^{-1})$	$1/\mathcal{R}_x$
Convolution	$x[n] * y[n]$	$X(z)Y(z)$	$\mathcal{R}_x \cap \mathcal{R}_y$

Common z -transform pairs

$x[n]$	$X(z)$	$\mathcal{R}_x$
$\delta[n]$	1	$\forall z$
$\delta[n - n_0]$	z^{-n_0}	Possibly $\forall z$
$u[n]$	$\frac{1}{1 - z^{-1}}$	$ z > 1$
$-u[-n - 1]$	$\frac{1}{1 - z^{-1}}$	$ z < 1$
$a^n u[n]$	$\frac{1}{1 - az^{-1}}$	$ z > a $
$-a^n u[-n - 1]$	$\frac{1}{1 - az^{-1}}$	$ z < a $
$na^n u[n]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z > a $
$-na^n u[-n - 1]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z < a $

TABLE A3. SOME COMMON WINDOWS FOR FIR FILTER DESIGN

Ideal lowpass filter, $n=0, 1, 2, \dots, N$ (Filter order given by N)

$$h_d[n] = \frac{\sin(\omega_c(n - N/2))}{(n - N/2)\pi}$$

Rectangular	$w[n] = \begin{cases} 1 & 0 \leq n \leq N \\ 0 & \text{else} \end{cases}$
Hanning	$w[n] = \begin{cases} 0.5 - 0.5 \cos\left(\frac{2n\pi}{N}\right) & 0 \leq n \leq N \\ 0 & \text{else} \end{cases}$
Hamming	$w[n] = \begin{cases} 0.54 - 0.46 \cos\left(\frac{2n\pi}{N}\right) & 0 \leq n \leq N \\ 0 & \text{else} \end{cases}$
Blackman	$w[n] = \begin{cases} 0.42 - 0.5 \cos\left(\frac{2n\pi}{N}\right) + 0.08 \cos\left(\frac{4n\pi}{N}\right) & 0 \leq n \leq N \\ 0 & \text{else} \end{cases}$
Kaiser	$w[n] = \frac{I_0\left(\beta \sqrt{1 - \left(\frac{n - \alpha}{\alpha}\right)^2}\right)}{I_0(\beta)} \quad 0 \leq n \leq N \quad \alpha = N/2$ where $\beta = \begin{cases} 0.1102(A - 8.7), & A > 50 \\ 0.5842(A - 21)^{0.4} + 0.07886(A - 21), & 21 \leq A \leq 50 \\ 0, & A < 21 \end{cases}$ $N = \begin{cases} (A - 7.95)/(2.285\Delta\omega), & A \geq 21 \\ 5.655/\Delta\omega, & A < 21 \end{cases}$

TABLE A4. COMPARISON OF COMMONLY USED WINDOWS

Type of Window (Filter length = $N+1$)	Peak Side-Lobe Amplitude (Relative)	Approximate Width of Main Lobe	Stopband Attenuation, $20\log_{10}\delta$ (dB)	Transition Width (Δf)
Rectangular	-13	$4\pi / (N+1)$	-21	$0.9/N$
Bartlett	-25	$8\pi / N$	-25	
Hanning	-31	$8\pi / N$	-44	$3.1/N$
Hamming	-41	$8\pi / N$	-53	$3.3/N$
Blackman	-57	$12\pi / N$	-74	$5.5/N$

TABLE A5. IIR FILTER DESIGN**TABLE A5.1. THE COEFFICIENTS IN THE SYSTEM FUNCTION OF A NORMALIZED BUTTERWORTH LOW PASS FILTER ($\Omega_c=1$) FOR ORDERS $1 \leq N \leq 8$**

N	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
1	1.0000							
2	1.4142	1.0000						
3	2.0000	2.0000	1.0000					
4	2.6131	3.4142	2.6131	1.0000				
5	3.2361	5.2361	5.2361	3.2361	1.0000			
6	3.8637	7.4641	9.1416	7.4641	3.8637	1.0000		
7	4.4940	10.0978	14.5918	14.5918	10.0978	4.4940	1.0000	
8	5.1258	13.1371	21.8462	25.6884	21.8462	13.1372	5.1258	1.0000

TABLE A5.2. CLOSED-FORM EXPRESSION FOR SOME USEFUL SERIES

$\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}$ $\sum_{n=0}^{N-1} na^n = \frac{(N-1)a^{N+1} - Na^N + a}{(1-a)^2}$ $\sum_{n=0}^{N-1} n = \frac{1}{2}N(N-1)$ $\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \quad a < 1$	$\sum_{n=0}^{\infty} na^n = \frac{a}{(1-a)^2} \quad a < 1$ $\sum_{n=0}^{N-1} n^2 = \frac{1}{6}N(N-1)(2N-1)$ $\sum_{n=N_1}^{N_2} a^n = \frac{a^{N_1-1} - a^{N_2}}{1-a}$
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TABLE A6. PARAMETERS OF ANALOG FILTER PROTOTYPES

Discrimination factor $d = \left[\frac{(1 - \delta_p)^{-2} - 1}{\delta_s^{-2} - 1} \right]^{1/2}$

Selectivity factor $k = \frac{\Omega_p}{\Omega_s}$

Butterworth	Order, $N \geq \frac{\log d}{\log k}$ System function, $H_a(s) = \prod_{k=0}^{N-1} \frac{-s_k}{s - s_k}$ where $s_k = \Omega_c e^{j(2k+1+N)\pi/(2N)}$
Chebyshev-I	Order, $N \geq \frac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)}$ System function, $H_a(s) = H_a(j0) \prod_{k=0}^{N-1} \frac{-s_k}{s - s_k}$ where $s_k = -\Omega_p \sinh[\sinh^{-1}(1/\epsilon)/N] \sin[(2k+1)\pi/(2N)]$ $+ j\Omega_p \cosh[\sinh^{-1}(1/\epsilon)/N] \cos[(2k+1)\pi/(2N)]$ $\epsilon = \sqrt{(1 - \delta_p)^{-2} - 1}$ $H_a(j0) = \begin{cases} 1, & \text{odd } N \\ (1 + \epsilon^2)^{-0.5}, & \text{even } N \end{cases}$
Chebyshev-II	Order, $N \geq \frac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)}$ System function, $H_a(s) = \prod_{k=0}^{N-1} \frac{s_k}{s'_k} \left(\frac{s - s'_k}{s - s_k} \right)$ where $s_k = \Omega_s \{ -\sinh[\sinh^{-1}(1/\epsilon)/N] \sin[(2k+1)\pi/(2N)]$ $+ j \cosh[\sinh^{-1}(1/\epsilon)/N] \cos[(2k+1)\pi/(2N)] \}^{-1}$ $s'_k = j\Omega_s / \cos((2k+1)\pi/(2N))$ $\epsilon = 1/\sqrt{\delta_s^{-2} - 1}$

Elliptic	$N \geq \frac{\log(16/d^2)}{\log(1/q)},$ $q = q_o + 2q_o^5 + 15q_o^9 + 150q_o^{13},$ $q_o = \frac{1}{2} \left[\frac{1 - (1 - k^2)^{1/4}}{1 + (1 - k^2)^{1/4}} \right]$
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TABLE A7. DISCRETE-TIME FOURIER ANALYSIS

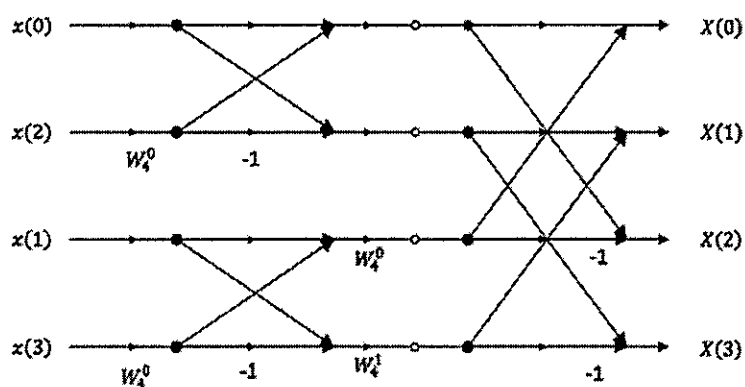
The discrete Fourier transform (DFT)

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}, \quad k = 0, 1, \dots, N-1$$

DFT in Matrix Format

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

The decimation-in-time FFT



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